

II B. Tech I Semester Supplementary Examinations, September - 2014

MATHEMATICS - III

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks

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- Prove that $\int_{-1}^1 p_m(x).p_n(x)dx = 0$ if $m \neq n$.
 - Prove that $(1-x^2)p_n^1(x) = n[p_{n-1}(x) - xp_n(x)]$
 - Show that when n is a positive integer
 - $J_{-n}(x) = (-1)^n J_n(x)$
 - $J_n(-x) = (-1)^n J_n(x)$ for +ve or -ve integers
 - Prove that $J_3(x) = \frac{8-x^2}{x^2} J_1(x) - \frac{4}{x} J_0(x)$
 - Show that the function given by $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at the origin.
 - Find the analytic function whose imaginary part is $f(x, y) = x^3y - xy^3 + xy + x + y$ where $z=x+iy$.
 - Evaluate $\int_C \frac{z+1}{z^2 + 2z + 4} dz$, where $C : |z+1+i| = 2$, using Cauchy's integral formula
 - Evaluate $\int_{-1+i}^{2+i} (x^2 + y^2 - ixy) dz$ along $y = x^2$
 - State and prove Laurent's theorem
 - Obtain all the Laurent expansions of the function $\frac{7z-2}{(z+1)z(z-2)}$ about $z = -2$
 - Determine the poles and the corresponding residues of $f(z) = \frac{2z+1}{z^2 - z - 2}$
 - Evaluate $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where $C : |z| = 4$.



7. a) Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

8. a) Show that the transformation $w = \frac{1}{z}$ maps a circle to a circle (or) to a straight line if the former goes through the origin.

b) Find the image of $1 < |z| < 2$ under the transformation $w = 2iz + 1$



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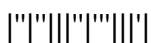
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1. a) Prove that  $\int_{-1}^1 x^m p_n(x) dx = 0$  if  $m < n$  using Rodrigue's formula.  
b) Prove that  $(2n+1)p_n(x) = p_{n+1}'(x) - p_{n-1}'(x)$
2. a) Prove that  
i)  $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{\sin(x)}{x} - \cos(x) \right]$   
ii)  $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{3-x^2}{x^2} \sin(x) - \frac{3}{x} \cos(x) \right]$   
b) Prove that  $\int J_3(x) dx = C - J_2(x) - \frac{2}{x} J_1(x)$
3. a) If  $W = \phi + i\psi$  represents the complex potential for an electric field and  $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$ , determine the function  $\phi$ .  
b) If  $f(z) = u + iv$  is an analytic function of  $z$  and  $u - v = e^x(\cos(x) - \sin(y))$  find  $f(z)$  in terms of  $z$ .
4. a) Determine  $\int_C \frac{z+4}{z^2+2z+5} dz$  where  $C$  is the circle.  
i).  $|z|=1$       ii).  $|z+1+i|=2$       iii).  $|z+1+i|=2$   
b) Evaluate  $\int_C \frac{z^2-z+1}{z-1} dz$  where  $C$  is the circle  
i).  $|z|=2$       ii).  $|z|=\frac{1}{2}$
5. a) Find the Taylor's expansion for the function  $f(z)$  where  $f(z) = \frac{1}{(1+z)^2}$  with center at  $z = -i$ .  
b) Expand  $f(z) = \frac{1}{z^3 - z - 6}$  about i).  $z = -1$  ii).  $z = 1$



6. a) Evaluate  $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$  where C is the circle  $|z|=3/2$  using residue theorem.

b) Evaluate  $\int_C \frac{e^z}{z^2+1} dz$  over the circular path  $|z|=2$ .

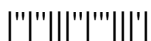
7. a) Show that by the method of contour integration,

$$\int_0^{\infty} \frac{\cos(mx)}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3} (1-ma)e^{-ma}, \quad a > 0, \quad b > 0$$

b) Determine the poles and residues at each pole of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

8. a) Find the image of a circle  $|z-2i|=2$  under the transformation  $w=1/z$ .

b) Find the image of an infinite strip  $0 < y < 1/2$  under the transformation  $w=1/z$ .



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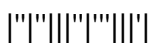
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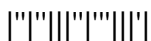
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- Prove that  $\int_{-1}^1 [p_n(x)]^2 dx = \frac{2}{2n+1}$
    - State and prove Rodrigue's formula.
  - Prove that  $\frac{d}{dx}(xJ_n J_{n+1}) = x(J_n^2 - J_{n+1}^2)$
    - Express  $J_5(x)$  in terms of  $J_0(x)$  and  $J_1(x)$
  - Show that the function  $f(z) = \begin{cases} \frac{x^3 y^3 (y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$  is not differentiable at the origin.
    - If  $u + iv = \frac{2 \sin(2x)}{e^{2y} + e^{-2y} - 2 \cos(2x)}$  and  $f(z) = u + iv$  is an analytic function of  $z$ , find  $f(z)$  in terms of  $z$ .
  - Evaluate  $\int_C (x^2 + ixy) dz$  from A(1,1) to B(2,8) along
      - the straight line AB
      - The curve  $x=t, y=t^3$
    - Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the paths i).  $y = x$  ii).  $y = x^2$
  - Find the Laurent's series of  $\frac{7z-2}{(z+1)z(z-2)}$  in the annulus  $1 < |z+1| < 3$
    - Find the Taylor's expansion of  $f(z) = \frac{2z^3+1}{z^2+z}$  about the point  $z = i$ .
  - Find the poles and residues at each pole of  $\tanh(z)$ .
    - Evaluate  $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$  where C is the circle  $|z|=3/2$  using residue theorem



7. a) Prove by contour integration  $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$
- b) Evaluate  $\int_C \frac{e^{2z} + z^2}{(z-1)^5} dz$  where  $C: |z|=2$  using Cauchy's Residue theorem.
8. a) Find the image of the triangle with vertices at  $i, 1+i, 1-i$  in the  $z$ -plane under the transformation  $w=3z+4-2i$ .
- b) Find the image of the rectangle  $-\pi < x < \pi, 1/2 < y < 1$  under the transformation  $w=\sin(z)$ .



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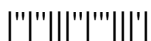
1. a) Using generating functions of Legendre's polynomial, prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$.
b) Express x^3 using Legendre's polynomials. $P_1(x)$ and $P_3(x)$.

2. a) Show that i) $\cos(x) = J_0 - 2J_2 + 2J_4$ ii) $\sin(x) = 2J_1 - 2J_3 + 2J_5$
b) Prove that $\frac{d}{dx}[J_n^2 + J_{n+1}^2] = \frac{2}{x}[2J_n^2 - (n+1)J_{n+1}^2]$ or
$$J_n J_n' + J_{n+1} J_{n+1}' = \frac{1}{x}[nJ_n^2 - (n+1)J_{n+1}^2]$$

3. a) Find the analytic function whose imaginary part is $f(x, y) = x^3 y - xy^3 + xy + x + y$ where $z = x + iy$
b) Prove that $\left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right] |\text{Real } f(z)|^2 = 2|f'(z)|^2$ where $w = f(z)$ is analytic

4. a) Find the analytic function whose imaginary part is $e^x(x \sin(x) + y \cos(y))$
b) Find the Principal value of $\left[\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right]^{1+i\sqrt{3}}$.

5. a) Evaluate $\int_C (x+y)dx + x^2 y dy$ from (0,0) to (3,9) i). along $y = x^2$ ii). along $y = 3x$
b) If $F(a) = \int_C \frac{3z^2 + 7z + 1}{z - a} dz$ where C is the circle $|z|=2$. Find the value of $F(1)$, $F(3)$, $F(1-i)$ and $F^{11}(1-i)$



6. a) Obtain the Laurent's expansion of the function $\frac{e^z}{(z-1)^2}$ in the neighborhood of its singular points and hence find its residue.
- b) Evaluate $\int_C \frac{\coth(z)}{z-i} dz$ where C is $|z|=2$
7. a) Show that $\int_0^{2\pi} \frac{d\theta}{a+b\sin(\theta)} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$ using residue theorem
- b) Evaluate by contour integration $\int_0^\infty \frac{dx}{(1-x^2)}$
8. a) Find the image of the infinite strip bounded by $x=0$ and $x=\pi/4$ under the transformation $w=\cos(z)$
- b) Find the image of infinite strips $1/4 < y < 1/2$ under the transformation $w=1/z$. Show the regions graphically.

